## Truth Tables

I. Negation (~: not) expresses the opposite truth value.

| $p$ | $\sim p$ |
| :---: | :---: |
| T | F |
| F | T |

II. Conjunction ( $\wedge$ : and) is true only III. Disjunction (V: or) is false only when both statements are true. when both statements are false.

| $p$ | $q$ | $p \wedge p$ |
| :---: | :---: | :---: |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |


| $p$ | $q$ | $p \vee q$ |
| :---: | :---: | :---: |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

IV. Conditional $(\rightarrow$ : if-then) is false only $V$. Biconditional $(\leftrightarrow$ : if and only if) is when the antecedent ( $1^{\mathrm{st}}$ ) is true and true only when the component the component ( $\left.2^{\text {nd }}\right)$ is false. statements have the same truth value.

| $p$ | $q$ | $p \rightarrow q$ |
| :---: | :---: | :---: |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |


| $p$ | $q$ | $p \leftrightarrow q$ |
| :---: | :---: | :---: |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

VI. Order to perform logic operators in truth tables:

- Parenthesis

○ ()

- Negation $\circ$ ~
- Conjunction o
$\Lambda$
- Disjunction o

V

- Conditional and
Biconditional
$\circ \rightarrow$ and $\leftrightarrow$
VII. Truth Table Examples:

1. $\sim p \wedge p$
2. $\sim(p \wedge q)$

| $\boldsymbol{p}$ | $\sim \boldsymbol{p}$ | $\sim \boldsymbol{p} \wedge \boldsymbol{p}$ |
| :---: | :---: | :---: |
| T | F | F |
| F | T | F |


| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $(\boldsymbol{p} \wedge \boldsymbol{q})$ | $\sim(\boldsymbol{p} \wedge \boldsymbol{q})$ |
| :---: | :---: | :---: | :---: |
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

3. $p \vee \sim q$

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\sim \boldsymbol{q}$ | $\boldsymbol{p} \mathbf{v} \sim \boldsymbol{q}$ |
| :---: | :---: | :---: | :---: |
| T | T | F | T |
| T | F | T | T |
| F | T | F | F |

4. $\sim(p \vee q)$

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $(\boldsymbol{p} \vee \mathbf{q})$ | $\sim(\boldsymbol{p} \vee \mathbf{q})$ |
| :---: | :---: | :---: | :---: |
| T | T | T | F |
| T | F | T | F |
| F | T | T | F |


| F | F | T | T |
| :--- | :--- | :--- | :--- |


| $F$ | $F$ | $F$ | $T$ |
| :--- | :--- | :--- | :--- |

5. $(p \wedge q) \vee r$

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\boldsymbol{r}$ | $(\boldsymbol{p} \wedge \boldsymbol{q})$ | $(\boldsymbol{p} \wedge \boldsymbol{q}) \mathbf{v} \boldsymbol{r}$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | F | T |
| T | F | F | F | F |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

6. $p \rightarrow(q \wedge r)$

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\boldsymbol{r}$ | $(\boldsymbol{q} \wedge \boldsymbol{r})$ | $\boldsymbol{p} \rightarrow(\boldsymbol{q} \wedge \boldsymbol{r})$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | T | T | T |
| T | T | F | F | F |
| T | F | T | F | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | F | F | T |
| F | F | T | F | T |



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7. Determine the truth value for $\sim p \wedge(\sim q \vee r)$ when $p$ is false, $q$ is true, and $r$ is false.
$\sim \mathrm{p} \wedge(\sim \mathrm{q} \vee \mathrm{r}) \quad$ Original statement
$\sim F \wedge(\sim T \vee F) \quad$ Original statement with truth values
$\sim \mathrm{F} \wedge(\mathrm{F} \vee \mathrm{F}) \quad$ Perform the negation in the parenthesis
$\sim F \wedge(F) \quad$ Finish the parenthesis
$\mathrm{T} \wedge(\mathrm{F}) \quad$ Perform the negation
False Perform the conjunction
8. Determine the truth value for $\sim p \vee \sim(q \vee r)$ when $p$ is false, $q$ is true, and $r$ is false.
$\sim p \vee \sim(q \vee r) \quad$ Original statement
$\sim$ F V ~(TVF) Original statement with truth values
$\sim$ F V $\sim(T) \quad$ Perform the parenthesis
TVF Perform the negations
True Perform the disjunction
9. Determine the truth value for $(\sim p \wedge q) \leftrightarrow \sim r$ when $p$ is false, $q$ is true, and $r$ is false.
$(\sim \mathrm{p} \wedge \mathrm{q}) \leftrightarrow \sim \mathrm{r} \quad$ Original statement
$(\sim \mathrm{F} \wedge \mathrm{T}) \leftrightarrow \sim \mathrm{F} \quad$ Original statement with truth values
$(\mathrm{T} \wedge \mathrm{T}) \leftrightarrow \sim \mathrm{F} \quad$ Perform the negation in the parenthesis
(T) $\leftrightarrow \sim \mathrm{F} \quad$ Finish the parenthesis

$$
\mathrm{T} \leftrightarrow \mathrm{~T} \quad \text { Perform the negation }
$$

True Perform the biconditional

Fall 2017

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