

Symmetry

I. Testing for Symmetry

Symmetry with Respect to:	Replacement	Result
y-axis	Replace x with -x	Equation unchanged
x-axis	Replace y with -y	Equation unchanged
origin	Replace x with -x and y with -y	Equation unchanged

Determine whether the following functions are symmetric with respect to the y -axis, x -axis, origin, or none of them.

1. $y = 3x^4 - 5x^2$

y-axis?

$$y = 3(-x)^4 - 5(-x)^2$$

$$y = 3x^4 - 5x^2$$

YES

x-axis?

$$-y = 3x^4 - 5x^2$$

$$-(-y) = -(3x^4 - 5x^2)$$

$$y = -3x^4 + 5x^2$$

NO

origin?

$$-y = 3(-x)^4 - 5(-x)^2$$

$$-(-y) = -(3x^4 - 5x^2)$$

$$y = -3x^4 + 5x^2$$

NO

Symmetric with respect to the y -axis.

2. $x^2 + y^2 = 25$

y-axis?

$$(-x)^2 + y^2 = 25$$

$$x^2 + y^2 = 25$$

YES

x-axis?

$$x^2 + (-y)^2 = 25$$

$$x^2 + y^2 = 25$$

YES

origin?

$$(-x)^2 + (-y)^2 = 25$$

$$x^2 + y^2 = 25$$

YES

Symmetric with respect to y -axis, x -axis, and origin.

3. $y = x^3 - 4x$

y-axis?

$$y = (-x)^3 - 4(-x)$$

$$y = -x^3 + 4x$$

NO
x-axis?

$$-y = x^3 - 4x$$

$$-(-y) = -(x^3 - 4x)$$

$$y = -x^3 + 4x$$

NO
origin?

$$-y = (-x)^3 - 4(-x)$$

$$-(-y) = -(-x^3 + 4x)$$

$$y = x^3 - 4x$$

YES
Symmetric with respect to the origin.
II. Testing for Even and Odd Functions.

Even and Odd Functions	Test Formulas	Symmetry
Even Function	$f(-x) = f(x)$	<i>y</i> -axis
Odd Function	$f(-x) = -f(x)$	origin

Determine whether the following functions are even or odd.

1. $f(x) = 2x^5 + 7x^3$

$$f(-x) = 2(-x)^5 + 7(-x)^3$$

$$f(-x) = -2x^5 + -7x^3$$

$$f(-x) = -(2x^5 + 7x^3)$$

Odd Function

2. $f(x) = -3x^2 - 10$

$$f(-x) = -3(-x)^2 - 10$$

$$f(-x) = -3x^2 - 10$$

Even Function



M-S4