

Completing the Square

1. Make sure the coefficient of x^2 is 1.
2. Take the constant to the other side.
3. Divide the coefficient of x by 2 and square the result, then add to both sides.
4. You just completed the square because now the left side of the equation is a perfect square trinomial.
5. Factor using the factoring formula for perfect square trinomials: $x^2 + 2xy + y^2 = (x + y)^2$ or $x^2 - 2xy + y^2 = (x - y)^2$
6. Take the square root of both sides.
7. Simplify.
8. Solve.

Example: $3x^2 - 18x - 48 = 0$

$$\begin{aligned} & 3x^2 - 18x - 48 = 0 \\ & \left(\frac{1}{3}\right)x^2 - 6x - 16 = 0 \end{aligned} \quad \text{Step 1}$$

$$\begin{aligned} & x^2 - 6x - 16 = 0 \\ & x^2 - 6x - 16 + 16 = 0 + 16 \\ & x^2 - 6x = 16 \end{aligned} \quad \text{Step 2}$$

$$x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 16 + \left(\frac{-6}{2}\right)^2 \quad \text{Step 3}$$

$$\begin{aligned} & x^2 - 6x + 9 = 16 + 9 \\ & x^2 - 6x + 9 = 25 \end{aligned} \quad \text{Step 4}$$

$$\begin{aligned} \text{Step 5} & (x - 3)^2 = 25 \\ \text{Step} & \sqrt{(x - 3)^2} = \pm\sqrt{25} \end{aligned} \quad 6$$

$$\begin{aligned} & x - 3 = \pm 5 \\ & x - 3 + 3 = \pm 5 + 3 \\ & x = \pm 5 + 3 \end{aligned} \quad \text{Step 7}$$

$$x = 8, x = -2 \quad \text{Step 8 Answer}$$

Example: $x^2 - 2x - 2 = 0$

$$x^2 - 2x - 2 + 2 = 0 + 2 \quad \text{Step 2} \quad x^2 - 2x = 2$$

$$x^2 - 2x + \left(\frac{-2}{2}\right)^2 = 2 + \left(\frac{-2}{2}\right)^2 \quad \text{Step 3}$$

$$\begin{aligned} & x^2 - 2x + 1 = 2 + 1 \\ & x^2 - 2x + 1 = 3 \end{aligned}$$

$$\begin{aligned} \text{Step 4} & (x - 1)^2 = 3 \\ \text{Step 5} & \sqrt{(x - 1)^2} = \pm\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{Step 6} & x - 1 = \pm\sqrt{3} \\ & x - 1 + 1 = 1 \pm\sqrt{3} \end{aligned} \quad -1 = \pm\sqrt{3} \quad \text{Step 7}$$

$$x = 1 + \sqrt{3}, x = 1 - \sqrt{3}$$

Step 8 Answer

Fall 2017



M-C1